Inflationary cosmology with two-component fluid and thermodynamics

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Running head: Inflationary cosmology with two-component fluid and thermodynamics

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Abstract

We present a simple and self-consistent cosmology with a phenomenological model of quantum creation of radiation and matter due to decay of the cosmological constant Λ. The decay drives a non-isentropic inflationary epoch, which exits smoothly to the radiation-dominated era, without reheating, and then evolves to the dust era. The initial vacuum for radiation and matter is a regular Minkowski vacuum. The created radiation and matter obeys standard thermodynamic laws, and the total entropy produced is consistent with the accepted value. This paper is an extension of the model with the decaying cosmological constant considered in [1]. We compare our model with the quantum field theory approach to creation of particles in curved space.

1 Introduction

The aim of this paper is to extend the scenario of the evolution of the universe with smooth exit from inflation, and particle production at

the expense of the decaying cosmological constant, developed in paper [1]. In that paper thermodynamics and Einstein's equations led to an equation in which the Hubble rate H is determined by the particle number N. The model is completed by specifying the particle creation rate $\Gamma = \dot{N}/N$, which led to a second-order evolution equation for H. The evolution equation for H then has a simple exact solution, in which a non-adiabatic inflationary era exits smoothly to the radiation era, without a reheating transition. For this solution, there were given exact expressions for the cosmic scale factor, energy density of radiation and vacuum, temperature and entropy.

In the paper [2] we generalised the abovementioned results for the case of a scalar field φ interacting with radiation via the gravitational field, leading to cosmological evolution with smooth exit from inflation. Our particular task was to determine whether the theory formulated in terms of the scalar field φ could lead to any new results in comparison with the previous case of a decaying cosmological constant. We concluded that the presence of the scalar field φ in this model did not change considerably the physical results which have been obtained in the paper [1]. As result we argued that models with decaying cosmological constant Λ corresponding to a special case of the equation of state $p_\varphi = -\rho_\varphi$, describe adequately the smooth transition from inflation to radiation and give a reasonable prediction for the entropy of matter in the universe.

In this paper we continue to argue along the lines of the paper [1] generalising its results for a two-component cosmological fluid, i.e. radiation and matter. The aim of the paper is to build a scenario of overall evolution of the universe with smooth exit from inflation to the radiation dominated stage and then its further evolution to the matter-dominated universe.

The background of this paper is constituted by two ideas in physical cosmology. One of them is connected with the longstanding attempt to explain all matter in the universe

as produced by quantum creation from vacuum. This has been studied via quantum field theory in curved spacetime (see for example [3, 4, 5, 6, 7]). Most cosmological models exhibit a singularity which presents difficulties for interpreting quantum effects, because all macroscopic parameters of created particles are infinite there. This leads to the problem of the initial vacuum (see discussion in [1]). One attempt to overcome these problems is via incorporating the effect of particle creation into Einstein's field equations. For example, in the papers of the Brussels group [8], the quantum effect of particle creation is considered in the context of the thermodynamics of open systems, where it is interpreted as an additional negative pressure, which emerges from a re-interpretation of the energy-momentum tensor. This effect is irreversible in the sense that spacetime can produce matter, leading to growth of entropy, while the reverse process is thermodynamically forbidden. These results were generalized in a covariant form in [9]. Our approach differs from that of [8, 9] in that we do not modify the field equations. Instead, we associate the source of created particles as a decaying cosmological constant Λ.

A number of decaying vacuum models has appeared in the literature (see [10, 11] and references cited there). A review of the different phenomenological models of evolution with variable cosmological term can be found in paper [12].

Inflationary models with fixed cosmological constant and cold dark matter have been successful in accounting for the microwave background and large-scale structure observations, while also solving the age problem (see [13]). However, these models are challenged by the reduced upper limits on Λ arising from the Supernova Cosmology Project (see [14]), and also by the long-standing problem of reconciling the very large early-universe vacuum energy density with the very low late-universe limits [11].

One resolution of these problems is a decaying cosmological constant Λ which is treated as a dynamical parameter. This approach was typical for the quantum field theorists for many years (see for example [15]). Many potential sources of fluctuating vacuum energy have been identified which were to give rise to a negative energy density which grows with time, tending to cancel out any pre-existing positive cosmological term and drive the net value of Λ toward zero. Processes of this kind are among the most promising ways to resolve the longstanding cosmological 'constant' problem (see [16] for review). It is worth mentioning the recent paper of Parker [17] indicating an attempt to revive the idea of the cosmological constant as a purely quantum effect associated with the renormalization of the general relativistic action.

In ad hoc prescriptions, the functional form of $\Lambda(t)$ or $\Lambda(a)$ or $\Lambda(H)$ (where a is the scale factor and H is the hubble rate) is effectively assumed a-priori (see the review [12] where all known forms for Λ are listed). Typically, the solutions arising from ad hoc prescriptions for Λ are rather complicated, and moreover, it is often difficult to provide a consistent simple interpretation of the features of particle creation, entropy and thermodynamics.

In contrast to many other models, we propose a simple, exact and thermodynamically consistent cosmological history. The latter originates from a regular initial vacuum. Together with naturally defined asymptotic conditions for the number of created particles this leads to a simple expansion law and thermodynamic properties, and to a definite estimate for the total entropy in the universe. Since the exit from inflation to the radiation era is smooth, we avoid the problem of matching at the transition. A similar smooth evolution has been used in [1, 2, 18, 19, 20].

The choice of a as dynamical variable and the very simple form of $H(a)$ that meets the physical conditions, lead to elegant expressions for all parameters describing the radiation and decaying vacuum, and also to a physically transparent interpretation of these results, including the estimate of entropy.

We use units with $8\pi G$, c and k_{B} equal 1.

2 Model

Metrics and Matter

We consider a spatially flat FRW universe we has the metric

$$ds^2 = -dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right] , \tag{1}$$

containing a uniform two-component cosmological fluid, which is composed of non-interacting matter and radiation. The radiation has an energy density $\rho_r(t)$ and a pressure $p_r = \rho_r/3$ while the non-relativistic matter has an energy density $\rho_m(t)$ and pressure $p_m = 0$. (we assume that $\rho_m \ll \rho_r$, $n_m \ll n_r$ at high temperature

T, so that we can neglect $p_m = n_m T$ with respect to p_r).

The energy momentum tensor of these components correspondingly is

$$T^R_{\mu\nu} = \frac{1}{3}\rho_r(t)\left[4u_\mu u_\nu + g_{\mu\nu}\right] , \qquad T^M_{\mu\nu} = \rho_m(t) u_\mu u_\nu ,$$

Decaying vacuum

We also consider matter corresponding to the quantum vacuum energy, with energy momentum tensor

$$T^Q_{\mu\nu} \equiv \langle \widehat{T}^Q_{\mu\nu} \rangle = \left[\frac{\Lambda(t)}{8\pi G}\right] g_{\mu\nu} .$$

(We shall use units such that $8\pi G = 1 = c$ henceforth) The conservation equations $\nabla^\nu(T^R_{\mu\nu} + T^M_{\mu\nu} + T^Q_{\mu\nu}) = 0$ reduce to

$$\dot{\rho_r} + \dot{\rho_m} + 4H\rho_r + 3H\rho_m = -\dot{\Lambda} , \tag{2}$$

(which is a special form of the first law of thermodynamics), where $H = \dot{a}/a$ is the Hubble rate. We can rewrite this equation, introducing an enthalpy of radiation and matter

$$h = h_r + h_m; \quad h_r \equiv \rho_r + p_r = 4/3\,\rho_r; \quad h_m \equiv \rho_m + p_m = \rho_m$$

so that

$$\dot{\rho} + 3Hh = -\dot{\Lambda} , \tag{3}$$

where $\rho = \rho_r + \rho_m$. The equations (2) and (3) show how energy is transferred from the vacuum to the radiation and matter densities. This energy transfer can therefore be understood as creation of quanta of radiation and matter from the vacuum. Indeed, employing the extended form of the first law of thermodynamics suggested in [8]:

$$d(\rho V) + pdV - \frac{h}{n} d(nV) = 0 , \tag{4}$$

one can connect the evolution of ρ and p with the evolution of the total number of particles (both photons and massive particles) $N = nV$, where V is a comoving volume of the observable universe. Since (4) is equivalent to

$$\dot{\rho} + 3Hh = h\frac{\dot{N}}{N}, \tag{5}$$

comparing with (3) gives

$$\frac{\dot{N}}{N} = -\frac{\dot{\Lambda}}{h}\,. \tag{6}$$

Therefore in order to create matter or radiation we need $\dot{\Lambda} < 0$, i.e. Λ to decrease with time.

It is clear from this formula that if $\Lambda = const$ we have no particle production and the total number of particles is conserved. Since radiation and matter do not interact it is legitimate to assume in this case that radiation and matter evolve separately according to standard conservation law

$$\dot{\rho_r} + 4H\rho_r = 0, \qquad \dot{\rho_m} + 3H\rho_m = 0, \tag{7}$$

which in conjunction with

$$d(\rho_r V) + p_r dV - \frac{h_r}{n_r} d(n_r V) = 0\,, \quad d(\rho_m V) + p_m dV - \frac{h_m}{n_m} d(n_m V) = 0\,, \tag{8}$$

tell us that the number of photons $N_r = n_r V$ and the number of massive particles $N_m = n_m V$ are conserved separately, so that $N = N_r + N_m = constant$.

This result gives us a clear understanding that in the case of $\dot{\Lambda} = 0$ the initial vacuum for photons and massive particles (where $N_r = N_m = 0$) will be stable leading to no particle creation in the universe. By switching on the source $\dot{\Lambda} \neq 0$ in the right-hand side of the equations (2) and (3) we effectively switch on coupling of radiation and matter with gravitational field. In other words the evolving $\Lambda(t)$ acts as an interaction of radiation and matter with the gravitational field leading, according to (6) to creation of photons and massive particles from vacuum.

Field equations

The field equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T^R_{\mu\nu} + T^M_{\mu\nu} + T^Q_{\mu\nu}$$

are

$$\begin{aligned} 3H^2 &= \rho_r + \rho_m + \Lambda\,, && (9)\\ 2\dot{H} + 3H^2 &= -\frac{1}{3}\rho_r + \Lambda\,, && (10) \end{aligned}$$

and if both are satisfied then the energy conservation equation (2) follows identically.

Following [18], we use a as a dynamic variable instead of t, and consider the Hubble rate as $H = H(a)$ (in this case we cannot consider a = constant as a limiting case for a flat universe). Given $H(a)$, we have two equations (2) and (9) from which we can determine $\rho_r(a), \rho_m(a), \Lambda(a)$.

Combining (2) and (9) one finds

$$h(a) = -\,a\,[H^2(a)]', \tag{11}$$

where primes denote d/da. If we differentiate (10)with respect to time and equate with (2), upon substitution for ρ_m from (10) one finds an equation for ρ_r. From this and (10) we also find an equation for ρ_m

$$\rho_r(a) = 3H^2(a) - R(a) + 3\Lambda(a), \qquad \rho_m(a) = R(a) - 4\Lambda(a), \tag{12}$$

where

$$R(a) = 3a[H^2(a)]' + 12H^2(a) \tag{13}$$

is a scalar curvature for metric (1). If we consider $a = 0$ as an initial point of the evolution of the universe, it is reasonable to assume that the beginning of the evolution is a vacuum for radiation and matter, i.e.

$$\rho_r(0) = 0\,, \qquad \rho_m(0) = 0,$$

which gives an initial condition $\Lambda(0) = 3H_0^2$, $H_0 = H(0)$. Obvious physical conditions for the energy densities are

$$\rho_r(a) \geq 0\,, \qquad \rho_m(a) \geq 0.$$

Using these conditions, equations (12) we can obtain the restriction for Λ which holds for any a:

$$a\,[H^2(a)]'(a) \leq \Lambda(a) - 3H^2(a) \leq \frac{3}{4}\,a\,[H^2(a)]'\,. \tag{14}$$

Assuming, that $a \geq 0$, and $H(a) \geq 0$ (i.e. have expansion of the universe), one can conclude, that this inequality makes sense only if

$$H'(a) \leq 0\,,$$

i.e. H is a decreasing function of a with the initial value H_0. Since $\Lambda(0) = 3H_0^2$, we have that

$$\Lambda(a) - 3H^2(a) \leq 0\,,$$

i.e. $\Lambda(a)$ is a decreasing function. Therefore we can deduce from (14) the formula for $\Lambda(a)$:

$$\Lambda(a) = \gamma(a)/2\,a\,[H^2(a)]' + 3H^2(a) \tag{15}$$

where $\gamma(a)$ is a continuous function with the range of values

$$\frac{3}{2} \leq \gamma(a) \leq 2\,.$$

On substituting (15) into (12) one finds

$$\rho_r(a) \;=\; \frac{6 - 3\,\gamma(a)}{2}\,(-a[H^2(a)]')\,, \quad \rho_m(a) \;=\; \frac{4\,\gamma(a) - 6}{2}\,(-a[H^2(a)]'). \tag{16}$$

These equations satisfy the physical conditions placed on the energy densities and it follows that if $\gamma = 3/2$, $\rho_m \equiv 0$, i.e. we have a universe containing pure radiation; and *vice versa*, and if $\gamma = 2$, $\rho_r \equiv 0$, i.e. we have only matter in the universe.

Now we have the formulas for ρ_r, ρ_m and Λ in terms of $H(a)$ and $\gamma(a)$. The next step hence is to make some reasonable assumptions about them.

The form of $\gamma(a)$

In order to make our model consistent with the fact that at present state of the universe radiation and matter evolve adiabatically, and matter dominates radiation, i.e. according to standard red shift law $\rho_r \sim a^{-4}$ and $\rho_m \sim a^{-3}$, we assume that at the late stage of evolution

$$\frac{\rho_r(a)}{\rho_m(a)} = \frac{3(2 - \gamma(a))}{2(2\gamma(a) - 3)} = \frac{a_m}{a},$$

where a_m is the value of the scale factor at time when the density of radiation and matter are equal ($a_m \approx 10^{-1} a_{decoupling}$). From this formula one can find $\gamma(a)$ for $a \gg a_m$:

$$\gamma(a) = \frac{6\,(1 + a/a_m)}{4 + 3a/a_m} \tag{17}$$

An amazing feature of this formula is that it adequately describes a smooth evolution of γ from $a = 0$ to $a = \infty$. Indeed $\gamma(0) = 3/2$, and $\lim_{a\to\infty} \gamma(a) = 2$. This result is in a coherence with the obvious physical expectation that the universe is dominated by radiation near the beginning of expansion, and the universe is dominated by matter at the late stage of its evolution. In other words the formula (17) gives us a simple form for $\gamma(a)$ for all values of a.

Assuming now that the simplest form for $\gamma(a)$ for any a is given by (17), and plugging (17) into (16) and (15) we find that

$$\rho_r(a) = \left[\frac{3}{4 + 3a/a_m}\right] \left[-a\,[H^2(a)]'\right], \quad \rho_m(a) = \frac{a}{a_m}\,\rho_r\,, \tag{18}$$

$$\Lambda(a) = \left[\frac{3\,(1 + a/a_m)}{4 + 3a/a_m}\right] \left[a\,[H^2(a)]'\right] + 3\,H^2(a). \tag{19}$$

3 The law of evolution for H(a)

The Form of H(a) in the vicinity a=0

To write down the above equations in a form which depends purely on the scale factor a, we need to make predictions about possible form of $H(a)$. To do this we bring into account two physical assumptions about the nature of the evolution of the universe in the vicinity $a = 0$ and when $a \to \infty$. It is by examining the behaviour of $H(a)$ in the light of these two assumptions we can predict its possible form.

Let us consider the case when $a \to 0$. In this case we can neglect $a/a_m \ll 1$ with respect to all O(1)- quantities in the expressions (17), (18), and (19) (i.e. $\gamma \approx 3/2$, corresponding to a purely radiation dominated universe as $a \to 0$).

Substituting (19) and (11) into (6) one obtains the equation for N when $a \to 0$:

$$\frac{N'}{N} = \frac{3}{4}\left[\frac{5}{a} + \frac{H'}{H} + \frac{H''}{H'}\right]\,,$$

which integrates to

$$N = A\left(-a^5 [H^2]'\right)^{3/4}\,, \tag{20}$$

where A is a constant. This expression for N can be rewritten as an equation for H:

$$\frac{d}{da} H^2(a) = -\frac{1}{2A}\,\frac{N^{4/3}(a)}{a^5}\,. \tag{21}$$

We require that initially there is a standard Minkowski vacuum for radiation, so that $N(a) \to 0$ and $n(a) \to 0$ as $a \to 0$. This implies the limiting behaviour

$$N(a) \sim a^{\alpha}\,, \quad \alpha > 3\,, \qquad \text{as} \quad a \to 0\,. \tag{22}$$

It then follows from equations (21) and (22) that [3]

$$H(a) \to \ \text{constant}\,, \qquad \text{as} \quad a \to 0\,. \tag{23}$$

$H(a)$ in the vicinity of exit from inflation

This form of the asymptotic behavior of H near $a = 0$ tells us that we have a type of expansion *a la* inflation. It cannot be an exact exponential inflation (with $H =$ constant) for al a, because in this case we would have $\rho_r = \rho_m \equiv 0$, $\Lambda =$constant, i.e. there would be an eternal stable false vacuum universe with no production of radiation and matter. Since the underlying motivation of our model is to obtain the observable figures for the energy-density and entropy of radiation and matter in the universe, one must assume that an initially inflationary universe will evolve into a present-state universe. Since inflation is usually understood as an expansion with acceleration, i.e. with $\ddot{a} > 0$, or $H + aH' > 0$, and at present we have decelerated phase, there must be exit from inflation such that $\ddot{a} = 0$, or $H_e = -a_e H'_e, H_e = H(a_e)$. This makes possible to calculate the ρ_r, ρ_m, Λ at $a = a_e$ in terms of H_e:

$$\rho_r(a_e) \;=\; \frac{6}{4 + 3a_e/a_m}\, H_e^2 \approx \frac{3}{2}\, H_e^2\,, \tag{24}$$

$$\rho_m(a_e) \;=\; \frac{6\, a_e/a_m}{4 + 3a_e/a_m}\, H_e^2 \approx \frac{3}{2}\,\frac{a_e}{a_m}\, H_e^2 = \frac{a_e}{a_m}\, \rho_r(a_e) \ll \rho_r(a_e) \tag{25}$$

$$\Lambda(a_e) \;=\; -\frac{6\,(1 + a_e/a_m)}{4 + 3a_e/a_m}\, H_e^2 + 3\, H_e^2 \approx \frac{3}{2} H_e^2. \tag{26}$$

(We assumed here $a_e \approx 10$ cm, $a_m \approx 10^{24}$ cm, and neglected $a_e/a_m \approx 10^{-23}$ with respect to all quantities of the order $O(1)$). Comparing (24) and (25) one can see that the creation of matter during the inflationary stage is damped with respect to the creation of photons with the amplitude 10^{-23}, the universe is therefore dominated by radiation at this stage with little matter.

Assuming now that created radiation is a black-body radiation, one can estimate the total number of photons created at exit from inflation. Using (24) for a standard calculation of entropy (see [18]) one can show, that $N_r(a_e) \approx 10^{88}$.

$H(a)$ for large a

In order to find an asymptotic for $H(a)$ for large a we employ again the assumption about an adiabatic evolution of radiation and matter at present, i.e. $\rho_r = \alpha a^{-4}$ and $\rho_m = \beta a^{-3}$. Using the formula (18) one can obtain an equation for $H^2(a)$ for large a:

$$\frac{d}{da} H^2(a) = -\left[\frac{4a_m + 3a}{3aa_m}\right] \frac{\alpha}{a^4} = -\frac{\alpha}{3a_m}\left[\frac{4a_m}{a^5} + \frac{3}{a^4}\right]$$

[3] It is occurring that the De Sitter stage of the evolution of the universe appears inevitably for any value of $3/2 \le \gamma \le 2$ taken as $a \to 0$, because, as follows from (18) and (19), Λ dominates evolution. One can show formally, that taking $\gamma =$ constant in (19), one can find that $H^2(a) \sim a^{(2\alpha-6)/\gamma} + C$, so that $H(a) \sim C$ as $a \to 0$ regardless of the value of γ.

which integrates to

$$H^2(a) = \frac{\alpha}{3a_m}\left[\frac{a_m}{a^4} + \frac{1}{a^3}\right]$$

(We disregard an arbitrary constant of integration because of an obvious condition for an open universe that $H \to 0$ as $a \to \infty$).

Matching of two asymptotics for $H(a)$

We have arrived to two asymptotic formulas for $H^2(a)$:

$$H^2(a) \sim \quad \text{const as} \quad a \to 0, \quad \text{and} \quad H^2(a) \sim \frac{a_m}{a^4} + \frac{1}{a^3} \quad \text{as} \quad a \to \infty$$

Our intention now is to combine these two asymptotics in order to find a formula for $H(a)$ which is valid for all a and describes a smooth transition from inflation to the adiabatic phase of expansion contingent upon independent evolution of radiation and matter.

In accordance with the method of the papers [1, 2], we try the formula for $H(a)$ in the following form:

$$H^2(a) = C\left[\frac{a_m}{(b^q + a^q)^n} + \frac{1}{(d^p + a^p)^m}\right] \tag{27}$$

where $q \cdot n = 4$ and $p \cdot m = 3$, as suggested by the asymptotic behaviour of $H(a)$ examined above.

We can easily fix the constants b^q and d^p by imposing the condition for the exit from inflation $2H_e^2 = -a_e(H_e^2)'$ in (27). We obtain that

$$b^q = a_e^q, \qquad d^p = a_e^p/2.$$

(It is interesting to note, that the condition for exit contains only the products qn and pm.)

The final choice of q, n, p, m we will base on the requirement to have at $a \to \infty$ the corrections to both leading terms in (27), i.e. $1/a^4$ and $1/a^3$ to be $O(1/a^8)$ and $O(1/a^6)$, i.e. we take $q = 4, n = 1, p = 3, m = 1$. The constant C in (27) can be found from equating both sides at $a = a_e$, so that finally (compare with a similar result from [20] and [21])

$$H^2(a) = H_e^2\,\frac{6a_m}{3a_m + 4a_e}\left[\frac{a_e^4}{a_e^4 + a^4} + \frac{2a_e^4}{a_m(a_e^3 + 2a^3)}\right] \tag{28}$$

In terms of the dimensionless variable $x = a/a_e$ (28) can be rewritten in the form:

$$H^2(x) = H_e^2\,\frac{2}{1 + 4/3\mu}\left[\frac{1}{1 + x^4} + 2\mu\frac{1}{1 + 2x^3}\right] \tag{29}$$

where $\mu = a_e/a_m \approx 10^{-23}$.

This formula describes the overall expansion of the universe starting from inflation with $H(x = 0) = \sqrt{2}H_e$. For $x < 1$ the asymptotic for H^2 has a form

$$H^2 = \frac{2H_e^2}{1 + 4/3\mu}\left[1 + 2\mu - 4\mu x^3 - x^4 + \ldots\right]$$

At exit $H(a_e) = H(x = 1) = H_e$. For $x > 1$ the asymptotic for H^2 is

$$H^2 = \frac{2H_e^2}{1 + 4/3\mu}\left[\frac{\mu}{x^3} + \frac{1}{x^4} - \frac{1}{2}\frac{\mu}{x^6} - \frac{1}{x^8} + \ldots\right].$$

One can obtain an estimate for H^2 at $a = a_m$ (the time when the matter and radiation energy densities are equal), or $x = \mu^{-1}$:

$$H_m \equiv H(x = \mu^{-1}) \approx 2H_e\mu^2$$

4 Dynamics of radiation and matter

Calculation of ρ_r, ρ_m and Λ

All further calculations will involve the expression

$$-a[H^2(a)]' = -x\frac{d[H^2(x)]}{dx} \equiv \frac{8H_e^2}{1+4/3\mu}\,\omega(x)$$

where

$$\omega(x) = \left[\frac{x^4}{(1+x^4)^2} + 3\mu\frac{x^3}{(1+2x^3)^2}\right]$$

such that

$$\omega(x=0) = 0, \qquad \omega(x=1) = \frac{1}{4}(1+4/3\mu).$$

with the asymptotics for the inflationary period, $x < 1$

$$\omega(x) \approx 3\mu x^3 + x^4 - 12\mu x^6 - 2x^8 + \dots\,,$$

and the asymptotic for expansion after inflation, $x > 1$

$$\omega(x) \approx \frac{3}{4}\frac{\mu}{x^3} + \frac{1}{x^4} - \frac{3}{4}\frac{\mu}{x^6} - \frac{2}{x^8} + \dots\,.$$

It is easy to obtain from here

$$\omega(x = \mu^{-1}) \approx \frac{7}{4}\mu^4.$$

The expressions for ρ_r, ρ_m and Λ have the form

$$\rho_r(x) = \frac{8H_e^2}{1+4/3\mu}\left[\frac{3}{4+3x\mu}\right]\omega(x)\,, \quad \rho_m(x) = x\mu\,\rho_r(x), \tag{30}$$

$$\Lambda(a) = -\frac{8H_e^2}{1+4/3\mu}\,\frac{3(1+x\mu)}{4+3x\mu}\,\omega(x) + 3H^2(x). \tag{31}$$

The leading terms in the asymptotics of these expressions for $x < 1$ have the form

$$\rho_r(x) \approx \frac{6H_e^2}{1+4/3\mu}\left[3\mu x^3 + (1 - 9/4\mu^2)x^4\right], \quad \rho_m(x) = x\mu\,\rho_r(x)\,, \tag{32}$$

$$\Lambda(a) \approx \frac{6H_e^2}{1+4/3\mu}\left[1 + 2\mu - 7\mu x^3 - (2+3/4\mu)x^4\right] \tag{33}$$

For the period from the exit from inflation to the time when the matter and radiation energy densities are equal i.e. for $1 < x \leq \mu^{-1}$ one finds

$$\rho_r(x) \approx \frac{24H_e^2}{1+4/3\mu}\left[\frac{1}{4+3x\mu}\right]\left[\frac{3}{4}\frac{\mu}{x^3} + \frac{1}{x^4}\right], \qquad \rho_m(x) = x\mu\,\rho_r(x) \tag{34}$$

$$\Lambda(x) \approx \frac{6H_e^2}{1+(4/3)\mu}\left\{-\frac{1+x\mu}{1+(3/4)x\mu}\left[\frac{3}{4}\frac{\mu}{x^3} + \frac{1}{x^4} - \frac{3}{4}\frac{\mu}{x^6}\right] + \left[\frac{\mu}{x^3} + \frac{1}{x^4} - \frac{1}{2}\frac{\mu}{x^6}\right]\right\}. \tag{35}$$

On substituting $x = \mu^{-1}$ one gets

$$\rho_r(a = a_m) = \rho_m(a = a_m) = \rho_r(x = \mu^{-1}) \approx 6H_e^2\mu^4 = 6H_e^2 \left[\frac{a_e}{a_m}\right]^4, \tag{36}$$

$$\Lambda(a = a_m) = \Lambda(x = \mu^{-1}) \approx 6H_e^2\,\frac{15}{7}\,\mu^7 = \frac{15}{7}\,\left[\frac{a_e}{a_m}\right]^7.$$

From the time when the energy densities were equal till the present and later (the matter dominated period) i.e. for $1 \ll \mu x \ll x$ then we find the leading terms are:

$$\rho_r(x) \approx 6H_e^2\frac{1}{x^4}, \qquad \rho_m(x) \approx 6H_e^2\frac{\mu}{x^3}, \qquad \Lambda(x) \approx \frac{16}{9}\frac{\mu}{x^5}.$$

These equations describe the standard red shift law $\rho_r \sim a^{-4}$
and $\rho_m \sim a^{-3}$ and show the adequacy of our choice for the form of $H^2(a)$ above

Particle Numbers and specific entropy

Solving, by integration, the equations for N_r and N_m which follow from (8)

$$\frac{N_r'}{N_r} = \frac{3}{4}\left[\frac{\rho_r'}{\rho_r} + \frac{4}{a}\right], \qquad \frac{N_m'}{N_m} = \left[\frac{\rho_m'}{\rho_m} + \frac{3}{a}\right], \tag{37}$$

one can get obvious expressions

$$\frac{N_r(a)}{N_r(a_e)} = \left[\frac{\rho_r(a)}{\rho_r(a_e)}\right]^{3/4}\left[\frac{a}{a_e}\right]^3, \qquad \frac{N_m(a)}{N_m(a_e)} = \left[\frac{\rho_m(a)}{\rho_m(a_e)}\right]\left[\frac{a}{a_e}\right]^3. \tag{38}$$

One can easily determine $N_r(a_e) = N_r(x = 1)$ from $\rho_r(a_e)$ using e.g. the results of [18]. Finally

$$N_r(x) = N_{r\,e}\left[\frac{16\omega(x)}{4 + 3\mu x}\right]^{3/4} x^3 \tag{39}$$

Figure 1 shows the evolution of $y(x) \equiv N_r(x)/N_{r\,e}$ over the expansion of the universe The number of quanta of radiation increases to a maximum value which is given by its limit as $x \to \infty$

$$N_{r\infty} = N_{re}2\sqrt{2}, \tag{40}$$

(we used $\mu \ll 1$).

We can not estimate $N_m(a_e)$ from $\rho_m(a_e)$, because we do not know the mass of the particles constituting matter. Instead of postulating this mass, we can connect $N_m(a_e)$ with $N_r(a_e)$ through the asymptotic condition for specific entropy s

$$s^* \equiv \lim_{x\to\infty}\frac{N_r(x)}{N_m(x)} \approx 10^8. \tag{41}$$

Using $\rho_m = (a/a_m)\rho_r$, one can easily prove that

$$\frac{N_m(x)}{N_{m\,e}} = \left[\frac{N_r(x)}{N_{re}}\right]^{4/3} = \frac{16\omega(x)}{4 + 3\mu x}\,x^4.$$

Taking limit $x \to \infty$ in the last formula one finds

$$N_{m\infty} = 4N_{me},$$

which together with (40) and (41) gives

$$N_{me} = \frac{s^{*-1}}{\sqrt{2}}\, N_{re}.$$

From the expressions (32) and (38) one can obtain the asymptotics for the number of particles:

$$N_r(a) \sim a^{21/4}, \qquad N_m(a) \sim a^7 \qquad \text{as} \quad a \to 0$$

For the photon creation rate (37) for $x = a/a_e < 1$ one finds from (32) (using $\mu = 10^{-23}$ and neglecting it with respect to O(1) terms) we find

$$\Gamma_r = \frac{\dot{N}_r}{N_r} = Ha\frac{N_r'}{N_r} \approx \frac{3}{4}H(x)\left(\frac{21\mu + 8x}{3\mu + x})\right).$$

In the limiting case $x \to 0$ we have

$$\Gamma_r(x \to 0) = \frac{21\sqrt{2}}{4}H_e$$

Then from (37) we find that

$$\Gamma_m = \frac{N_m'(a)}{N_m(a)} = \frac{4}{3}\frac{N_r'(a)}{N_r(a)} = H(x)\left(\frac{21\mu + 8x}{3\mu + x})\right) \tag{42}$$

and

$$\Gamma_m(x \to 0) = 7\sqrt{2}H_e.$$

For the specific entropy

$$s(a) \equiv \frac{N_r(a)}{N_m(a)}$$

one can easily obtain the equation for its evolution using (42):

$$\frac{\dot{s}}{s} = \left[\frac{\dot{N}_r(a)}{N_r(a)} - \frac{\dot{N}_m(a)}{N_m(a)}\right] = -\frac{1}{3}\frac{\dot{N}_r(a)}{N_r(a)} = -\frac{1}{3}\Gamma_r \leq 0.$$

The decrease of the s is connected with its definition and with the fact that N_m evolves faster to 0 than N_r. Since $\Gamma_r \to 0$ as $a \to \infty$ we have

$$\dot{s} \to 0 \text{ as } a \to \infty$$

One can find the formula for the evolution of $s(x)$

$$s(x) = s_e\left[\frac{16\omega(x)}{4 + 3\mu x}\right]^{-1/4} x^3, \qquad s_e \equiv \frac{N_{re}}{N_{me}} = s^*\sqrt{2}.$$

5 Particle creation in decaying cosmology and quantum field theory in curved space

In this final section we compare our phenomenological description of matter creation in an expanding universe with some known results from quantum field theory in curved space. Namely we compare the results for the N_r presented in a previous section with the number of particles N_q created from vacuum through quantum effects (see e.g. [4, 5, 7])

$$\frac{dN_q}{dt} = ca(t)^3 R^2(t) \qquad \text{or} \qquad \frac{dN_q}{da} = c\frac{a^2\, R^2(a)}{H(a)},$$

where R is the Ricci curvature given by the formula (13), and the constant "c" takes different values for different fields. This formula describes creation of massless non-conformal particles. We argued in the paper [7] that this formula can be integrated, so that the total number of particles created from the initial state at $x = 0$ to some x can be presented in the form ($x = a/a_e$):

$$N_q(x) = c\int_0^x \frac{x'^2\, R^2(x')}{H(x')} dx',$$

or

$$N_q(x) = N_{q\,e}\frac{\int_0^x \frac{x'^2\, R^2(x')}{H(x')} dx'}{\int_0^1 \frac{x'^2\, R^2(x')}{H(x')} dx'}. \tag{43}$$

where $N_{q\,e}$ is the number of particles created to $a = a_e$ ($x = 1$). If we assume now that our species of particles can be modelled by quanta, let say, of a minimally coupled scalar field, one can apply (43) for the universe which evolution is described effectively by the formula (29). In this case we tacitly assume that our phenomenological description of transfer of energy from decaying Λ to radiation and matter can be alternatively understood as quantum particle creation from vacuum in the metric (1) with the expansion law (29). This assumption makes sense since, as we have seen above, the most of particles is created during the inflationary stage, where the contribution from massive particles can be neglected.

Comparing two graphs for $y(x) \equiv N_r(x)/N_{r\,e}$ (formula (39) and $z(x) \equiv N_q(x)/N_{q\,e}$ (fomula (43) presented at the Fig. 1, we clearly see that the behavior of $z(x)$ and $y(x)$ is quite similar and the asymptotic values of z and y are of the same order:

$$z(\infty) = \frac{N_{q\infty}}{N_{q\,e}} = 1.113.. \qquad y(\infty) = \frac{N_{r\infty}}{N_{re}} = 2\sqrt{2},$$

Since $N_{q\,e}$ is a free parameter in th e formula (43) one can choose it from a natural condition that

$$N_{q\infty} = N_{r\infty}$$

so that

$$N_{q\,e} = 2.533..\, N_{r\,e}.$$

We can argue now that our phenomenological theory of production of radiation and matter via smooth evolution of the universe from inflation to the radiation-dominated stage and then to the dust stage, which is based on the form for the Hubble parameter (29), can be treated as a mimic of quantum particle creation from vacuum. This gives in a sense a microscopic justification for our phenomenological model.

Fig. 1 Graphs for

$$z(x) = \frac{N_q(x)}{N_{q\,e}}, \qquad y(x) = \frac{N_r(x)}{N_{r\,e}}$$

in dimensionless units where $x \equiv a/a_e$. The number of particles created through quantum effects N_q is given by the formula (43); $N_{q\,e}$ is its value at the time of exit from inflation. N_r and $N_{r\,e}$ are related to photon creation due to decay of Λ and are given by the formula (39). It is clearly seen the that the behavior of $z(x)$ and $y(x)$ is quite similar and the asymptotic values of z and y ar of the same order:

$$z(\infty) = \frac{N_{q\infty}}{N_{q\,e}} = 1.113.. \qquad y(\infty) = \frac{N_{r\infty}}{N_{re}} = 2\sqrt{2},$$

6 Conclusion

We have considered a simple and thermodynamically consistent scenario encompassing the decay of the vacuum, the creation of radiation and matter, and a natural smooth transition from inflationary to radiation- and then matter-dominated expansion. In order to treat all matter in the universe as created from a Minkowski vacuum, we impose the physical condition that the number of particles is zero in the initial vacuum state $a = 0$. Together with requiring finite particle production in the observable universe, this constrains the form of the Hubble

rate (29), and gives an inflationary universe with smooth exit to the radiation era and then to the dust era. Using (29) we calculated then the energy density for radiation and matter, their partlicle numbers and specific entropy for the overall evolution of the universe.

We argued that our phenomenological theory of production of radiation and matter via smooth evolution of the universe from inflation to the radiation-dominated stage and then to the dust stage, which is based on the form for the Hubble parameter (29), can be treated as a mimic of quantum particle creation from vacuum initial vacuum and give a microscopic justification for our model.

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